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# Radiation Resistance in Radial Transducer

E. SAWADO

## I. Introduction

The purpose of the present paper is to give an intuitive explanation for the characteristics of radial wave and to demonstrate that this mode has no cutoff below the critical frequency $\omega = \gamma(BH)^{1/2}$, where $\omega$ is the angular frequency, $\gamma = 1.76 \times 10^7$ ((oe sec)$^{-1}$ in CGS unit), $B = \mu_0(H+M)$ is the magnetic flux density, $H$ the magnetic field, $M$ the saturation magnetization. Ganguly and Webb, and others [1]–[3] presented an initial theory and experiments for magnetostatic surface wave transducers. They obtained some of the useful results for resistance of a microstrip due to radiation. Previous investigations have calculated dispersion characteristics [4] and characteristic impedance of composite microstrip slab structure [5]. These investigations conclude that microstrip excitation of magnetostatic surface wave has proven particularly convenient, because of strong coupling from electromagnetic waves to magnetostatic waves. It is easy to see that the lowest operating frequency of the Ganguly type delay line is $\gamma H$. Below this cutoff, no modes can exist. In view of the above, investigation of radial wave type delay line should produce useful developments in low frequency microwave (0.5 to 1.0 GHz) applications.

The system analyzed in this report is shown in Fig. 1. A transducer in the form of a fine wire is excited with an RF current which generate radial volume waves within the structure. The dc magnetic field is directed along the $z$ axis and also the fine wire is situated parallel to the $z$ axis. This mode propagates perpendicular to the magnetic biasing fields, guided by two parallel surfaces, and its energy is distributed within the medium.

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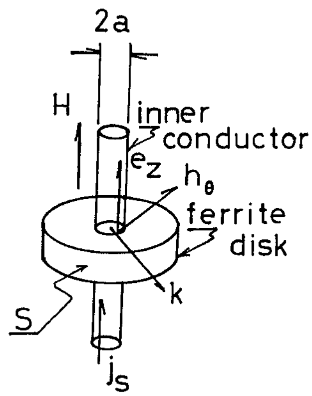


Fig. 1. Radial waveguide and an infinitely long wire as exciting antenna.

## II. Basic Theory

Coupled differential equations for the $z$ component of electric and magnetic fields are [6], [7].

$$\left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + \omega^2 \epsilon \mu_0 \mu_\perp\right) e_z + \omega \mu_0 \kappa / \mu \frac{\partial}{\partial z} h_z = 0 \quad (1)$$

$$\left(\nabla_\perp^2 + (1/\mu) \frac{\partial^2}{\partial z^2} + \omega^2 \epsilon \mu_0\right) h_z + \omega \epsilon \kappa / \mu \frac{\partial}{\partial z} e_z = 0 \quad (2)$$

where $\mu_0$ and $\epsilon$ are the vacuum permeability and dielectric constant, respectively, $\nabla_\perp^2$ is a differential operator $= \partial^2/\partial x^2 + \partial^2/\partial y^2$, and $\mu_\perp$ is the effective permeability given by

$$\mu_\perp = (\mu^2 - \kappa^2)/\mu \quad (3)$$

where $\mu$ and $\kappa$ are a diagonal and a nondiagonal component of the relative permeability tensor. These equations become two independent differential equations, when the fields are independent of $z(\partial/\partial z = 0)$. The assumption that $(\partial/\partial z = 0)$ is true only under two conditions: 1) when there are no energy leaks into the free space, and 2) when the thickness of the ferrite sheet is small compared with wavelength $\lambda$. We now express them in cylindrical coordinates

$$\left(\frac{1}{\sigma}\frac{\partial}{\partial \sigma}\left(\sigma \frac{\partial}{\partial \sigma}\right) + \omega^2 \epsilon \mu_0 \mu_\perp\right) e_z = 0 \quad (4)$$

$$\left(\frac{1}{\sigma}\frac{\partial}{\partial \sigma}\left(\sigma \frac{\partial}{\partial \sigma}\right) + \omega^2 \epsilon \mu_0\right) h_z = 0. \quad (5)$$

From (4), we have a solution for a radial argument in terms of the Bessel function of order 0 and the Hankel function of the second kinds of order 0

$$e_z = a_0 J_0(k\sigma) + b_0 H_0^{(2)}(k\sigma). \quad (6)$$

Substituting (6) into (4), we have

$$k^2 = \omega^2 \epsilon \mu_0 (\mu^2 - \kappa^2)/\mu. \quad (7)$$

This relation gives a dispersion of this system. The boundary condition on the $z$ component of electric field requires that

$$e_z = 0 \qquad \text{when } \sigma = a. \quad (8)$$

The $z$ component of the electric field satisfying the boundary

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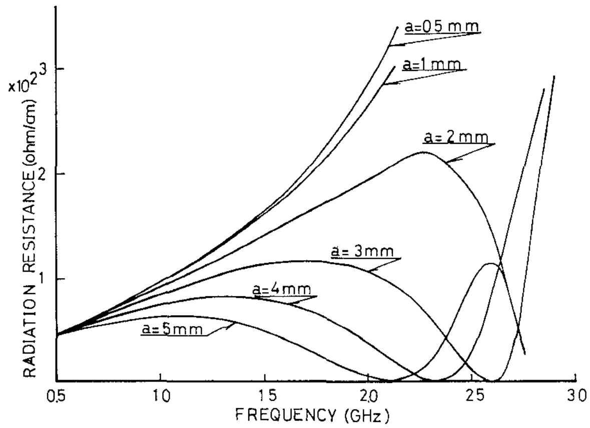


Fig. 2. Radiation resistance is plotted as a function of frequency for the radius of the inner conductor $a = 0.2$, 0.5, 1.0, 2.0, 3.0, and 4.0 m/m. Other parameters are biasing dc magnetic field $h = 500$ oe, saturation magnetization $4\pi M_s = 1800$ g.

condition is

$$e_z = a_0\left(J_0(k\sigma) - \frac{J_0(ka)}{H_0^{(2)}(ka)}H_0^{(2)}(k\sigma)\right). \tag{9}$$

As shown in Fig. 1, an infinitely long wire is situated parallel to the $z$ axis, so the current flowing into the wire excites the RF magnetic fields $h_\theta$. This means that the power flowing into the wire (RF current $j_s$) is equal to the radiating power. Since the energy dissipation is zero in the region of the field bounded by a surface $S$, the real part of complex Poynting vector is zero

$$\mathrm{Re}\left(\frac{1}{2}\int_S(-e_z h_\theta^*)\sigma\, d\theta\right) = 0 \tag{10}$$

where $h_\theta^*$ is the complex conjugate of $h_\theta$. From Maxwell's equation, we have a relation between $e_z$ and $h_\theta$

$$h_\theta = \left(\frac{-\mu}{\mu^2-\kappa^2}\right)\left(\frac{1}{j\omega\mu_0}\right)\frac{\partial^e z}{\partial\sigma}. \tag{11}$$

According to Ampere's law, we have a relation between the current $j_s$ and the amplitude $a_0$ of (6)

$$j_s = \int_0^{2\pi} h_\theta a\, d\theta. \tag{12}$$

This relation gives

$$a_0 = \frac{-j_s}{2\pi a}\frac{(\mu^2-\kappa^2)j\omega\mu_0}{\mu k}\left(J_1(ka) - \frac{J_0(ka)}{H_0^{(2)}(ka)}H_1^{(2)}(ka)\right)^{-1} \tag{13}$$

The radiation resistance $R_m$ is defined by the ratio the radiative power to the square of the current flowing into the metal cylinder

$$R_m = \frac{1}{16}\frac{\omega\mu_0}{k}\frac{\mu^2-\kappa^2}{\mu}(J_0(ka))^2. \tag{14}$$

In Fig. 2, the radiation resistance $R_m$ is plotted as a function of frequency. The important characteristics (shown in Fig. 2) are that this device transmits all signals from direct current up to some frequency near the critical frequency $\omega = \gamma(BH)^{1/2}$. This extremely wide bandwidth can be achieved by reducing the radius of the metallic wire $a$. Note that for $a = 0.5$ m/m, 1 to 2 thousand megahertz excitation bandwidths are possible. Owing to the above reasons, development of a suitable radial line should produce extremely wide bandwidths and magnetically tunable low frequency (0.5 to 1 GHz) microwave transducer.

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# Broad-Band Design of Improved Hybrid-Ring 3-dB Directional Couplers


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*Abstract*—**A broad-band design theory of an improved 3-dB hybrid-ring directional coupler is proposed and discussed. The synthesis of the improved broad-band hybrid-ring directional coupler starts by applying the concept of hypothetical port and generalizing the conventional hybrid-ring. The improved broad-band 3-dB hybrid-rings can be constructed very easily and their bandwidths are considerably wide, while the bandwidth of the reverse-phase hybrid-ring (one lambda ring) may be increased to approximately an octave, but it has not found wide acceptance because of its extreme difficulty of construction. The bandwidth of the improved broad-band 3-dB hybrid-ring directional coupler is 1.84 times as wide as the conventional rat race or hybrid-ring, extending from 0.747 to 1.253 in normalized frequency. Furthermore, the experimental verification has been achieved in microstrip network, and, hence, the validity of the design method proposed in this paper is confirmed. Although only the 3-dB hybrid-ring directional was considered here, the method itself is to be applicable to a hybrid-ring directional coupler with any degree of coupling.**


## I. Introduction

A hybrid-ring directional coupler is one of the fundamental components used in microwave circuits, which is recognized as a rat race ring when it is used for a 3-dB directional coupler with the normalized admittance of $1/\sqrt{2}$ on the whole circumference on the ring. The rat race or hybrid-ring directional coupler has the bandwidth of approximately 27.6 percent at the tolerance limits of the deviation of 0.43 dB for split and of 20 dB for the maximum return loss and isolation.

This paper describes two methods for broadening the bandwidth of the hybrid-ring while it operates as a 3-dB directional coupler. The broad-band design method proposed here was accomplished using CAD, in which the concept of hypothetical


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